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Bayesian Formulation of the Best of Liquid and Solid Reliability Methodology

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An approach to reliability prediction for liquid or solid rockets is presented. The approach is based on Bayesian statistical methods and incorporates existing reliability methodology. The Bayesian model provides a framework for incorporating various types of information and for regular revision and updating of predictions. It facilitates the calculation of confidence intervals for reliability and of required test numbers to achieve reliability goals.

Introduction

SINCE its inception, the rocket propulsion industry has concentrated primarily on designs that maximize payload capability in the context of specified, usually well-defined, mission and performance considerations, with costs and reliability of secondary concern. This is not to say that reliability was not considered to be important; it is often a critical factor. It is, however, much more difficult to predict than performance and weight, particularly at the design stage. As a result, reliability evaluation was based mainly on full system testing and operational outcomes.

The reliability levels that the industry has succeeded in achieving have come about through the use of large margins (safety factors) in part design and of a build-test-fail-fix approach. The success of the design as far as reliability was concerned thus depended on factors that were determined by engineering judgment derived from the company and engineer's background, past experience, historical data, trial and error, handbook statistics, and similar information sources.

Reliability analyses that were performed, if any, were usually based on oversimplified models and assumptions. In addition, process and operations-induced failures were seldom adequately addressed, if even included in the reliability analysis. As a result of these and many other difficulties, past reliability predictions have almost invariably overestimated the reliability of an operational system.

Over the past few years, this situation has been changing considerably, primarily as a result of the reliability and cost requirements specified for the Advanced Launch System (ALS), later called the National Launch System (NLS). For example, the ALS Phase A specification required that, for the space transportation engine (STE), a 0.99 reliability be achieved with 90% confidence. This greatly stimulated interest in analytical efforts toward early quantification of reliability, leading in turn to recognition of the need for careful and thorough identification of failure modes as well as the need for development of more realistic engineering and statistical models of failure mechanisms and their likelihood of occurrence. Moreover, funding is generally not available to achieve high reliability through an extensive development program.

Further impetus to early quantification and prediction is the importance of reliability in evaluating competing systems (liquids and solids) at the very earliest stages of design. An initial objective of the project that led to this paper was the development of a methodology for comparison of liquid and solid propulsion systems (or, in fact, any two candidate systems).

In approaching the problem of comparing liquids and solids, the first step was an extensive survey of liquid and solid contractors and others involved in reliability evaluation of rockets to determine the state of the art with regard to reliability prediction methodology. The results of the survey formed the basis of a proposed general approach to propulsion system reliability assessment, which is presented herein.

This paper begins with a brief look at historical data on success ratios of liquid and solid rockets. It is easily seen that it is not possible to choose between the two on the basis of success/fail data alone. The results of the reliability survey are discussed along with the recommended approach, featuring the perceived strengths of both liquids and solids. The recommended approach is based on Bayesian statistical methodology and requires that the liquid and solid approaches be reformulated in that framework. Some comments on the suggested statistical methodology that would be required to calculate reliabilities as recommended and the modeling and data requirements necessary for implementation of the approach are presented. Finally, a brief discussion is presented of the use of Bayesian statistical models in determining test requirements.

Historical Data

Many data bases have been compiled on outcomes of tests and operational firings of liquid engines and solid rocket motors. Results obtained in comparisons of the two depend on which data base is chosen,[1,2] the time frame selected, specific systems used in the comparison, and, of course, on the definition of success and failure in comparing reliabilities expressed in terms of success rates.

As an illustration of the statistical approach to comparison of two systems based on success ratio, we use a data base prepared recently by an independent source. The data are comparable to those used in Refs. 1 and 2. Failure is defined to be mission failure due to any cause, whether critical or not. The results for liquid engines used in the analysis given later are for Atlas, Titan, Delta, Centaur, Shuttle, and Saturn 5. For solid motors, data on Castor, Titan, Minuteman, Peacekeeper, and Shuttle are included. Estimated reliability R is calculated as $R = X/N$ in each case, where N is the number of attempted firings and X is the number of successes. In addition, the 90% lower confidence bound on reliability R_L, calculated by the standard statistical method based on exact binomial probabilities, will be given.

Note that a mixture of large and small propulsion elements is included for both liquids and solids. In addition, the data are operational data covering varying periods of time from program inception until early 1990. In this context, the bino-

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Table 1 Liquid vs solid rocket reliability and 90% confidence band, all data

	Successes, X	Failures	R	R_L
Liquids	1500	22	0.9855	0.9808
Solids	3035	13	0.9957	0.9938

Table 2 Liquid vs solid rocket reliability and 90% confidence band, small systems omitted

	Successes, X	Failures	R	R_L
Liquids	1363	17	0.9877	0.9829
Solids	1320	12	0.9910	0.9867

mial model is an approximation and provides only a comparison of average reliabilities.

The results that were obtained for a total of 1522 liquids and 3048 solids are shown in Table 1. In comparing these results for liquids and solids, we apply a standard statistical test, the chi-square test, which compares success ratios for the two types of systems. This result is a calculated value of chi-square of $\chi^2 = 13.86$, with 1 degree of freedom. For testing at the 5% level, the tabulated value of chi-square is 3.841. The hypothesis of independence is rejected, and we conclude that there is statistical evidence that liquids and solids differ with regard to reliability. Further, it leads to the conclusion that solids achieve the Phase A ALS reliability goal of 0.99 at 90% confidence.

If, however, we remove small units from the comparison to obtain a data set more nearly comparable to proposed ALS systems, a different result is obtained. The results of omitting Castors from solids and Centaur from liquids are shown in Table 2. For these data, the calculated value of χ^2 is 0.70, with 1 degree of freedom. This is not significant at the 5% (or any other) level, and we conclude that with this data base there is no evidence of a difference in reliability of large liquids and solids. Further, neither achieves the goal of 0.99 at 90% confidence.

We emphasize again that this is an approximate procedure. The binomial formulation requires independent observations, which are tenable, and constant reliability, which is not. A better procedure for comparing systems would be one that allowed for different reliabilities within groups. Procedures based on reliability growth models,[3-5] for example, would provide a reasonable alternative of this type, but the results would still be influenced by the choice of data base.

The point here is that the controversy regarding comparison of systems is unlikely to be settled by looking at historical data. Neither will reliability be evaluated and systems compared by testing alone. What is recommended is a methodology that utilizes all sources of information, including test data, for evaluating reliability of new and evolving systems. Historical data may most usefully be employed for this purpose.

Design for Reliability—Survey Results

Although outcomes analysis provides an overview and establishes a benchmark for reliability evaluation, it is an after-the-fact analysis that can basically be applied only to operational systems. It is a well-established axiom in quality control, however, that reliability cannot be inspected in, it must be designed in.

In assessing the current state of the art in the area of design for reliability, visits were made to a number of contractors in the propulsion industry and other interested parties. Overall, it was found that much has been done recently to improve the approach taken to reliability at the design stage in the propulsion industry. In principle, the approaches used or proposed for use in both the liquids and solids industries are similar, but there are distinct differences in emphasis, depth and extent of analysis, integration of efforts, and evaluation.

In general, the activities are organized by the contractors in the traditional sequence: conceptual design, preliminary design, detailed design, component testing, and engine/motor testing. All use the standard tools of reliability analysis,[4-6] including failure modes and effects analysis (FMEA), fault tree analysis (FTA), critical items lists (CIL), a deterministic analysis based on margins or safety factors, and, for critical parts/failure modes, some sort of probabilistic analysis. Significant differences exist, however, in how and when these tools are applied. This is also true of the activities that follow design, including development of test plans, data analysis, quality control programs, and so forth.

A recommended approach, intended to address this problem by providing a framework for a unified approach to reliability, is presented in this paper. This approach is based, in great part, on the strengths of the methodology currently proposed. The following are some specific comments on the perceived strengths and weaknesses of this methodology.

Evaluation of Liquids' and Solids' Approaches

Currently, all contractors recognize the importance of analysis as opposed to overdesign in assuring the reliability of propulsion systems. This is reflected to a greater or lesser extent in all of the approaches to reliability being used for ALS. Although many sound methodologies are being employed or proposed, there is a need for additional analysis in several areas.

All contractors recognize that testing alone cannot verify the high reliabilities required in ALS. Contractors are generally aware of the need for efficient experimental design techniques for use in obtaining the needed information, and efforts are under way in this area. All recognize that testing of individual failure modes, even when possible, is inadequate.

Also, it is recognized by most that many of the statistical and engineering models used in traditional reliability analyses, e.g., stress-strength models, are inadequate. Better probabilistic approaches are being developed, particularly in the liquids industry. A proper probabilistic analysis requires careful engineering modeling of the failure phenomenon, specification of statistical distributions for all conditions related to failure, and derivation, often by means of computer simulations, of the resultant failure distribution. Significant progress is being made in this area.[7] The liquids industry has the lead in development of this methodology and in adoption of these methods to some critical failures. Although a great deal of progress has been made in design studies, much remains to be done in implementation in actual applications.

A particular strength of the solids' approach is that the contractors specifically address process and operations-induced failure modes as well as design-induced failure modes. This is especially important since a significant proportion of all failures of current propulsion systems (about two-thirds, according to some estimates) are in these areas rather than in design. The solids are emphasizing extensive process control programs.

An additional strength of both liquids' and solids' approaches is the design-in reliability approach through the use of simultaneous engineering. The simultaneous engineering concept involves assignment of teams to provide input to the design on manufacturing and operational aspects as well as engineering inputs. The intent is to have all relevant disciplines represented on the team in order to effect the interaction of component design with production and operations.

Although the need for improvement in many areas is recognized by all contractors, it is apparent that not everyone is addressing or can address these needs adequately. Specifically, the following problem areas require additional attention.

1) More reliance on probabilistic analyses and less on deterministic methods and oversimplified models and assumptions.

2) A consistent method for assessing design by comparing allocated and predicted reliabilities.

3) Methods to deal with process and operations failures at the earliest design stages.

4) Human factors included in the analysis.

5) System models looked at realistically to determine if simplified assumptions such as series systems and complete independence are reasonable.

6) Need for a consistent, industry-wide method for predicting system reliability. (Lacking this, there is no meaningful way to compare systems prior to the acquisition of outcomes data.)

7) Statistical analyses need improvement in a number of areas. These again include unrealistic assumptions and some problems in experimental design, but also include improper or incomplete analysis of test data, lack of measures of precision on reliability estimates, and lack of confidence intervals and other important methodological tools.

8) Sensitivity analyses should be more widely used to identify the critical factors determining system reliability.

What is most seriously needed by all, however, is an approach incorporating sound engineering judgment with historical data, mathematical and statistical models, simulation, and experimental results that would provide reliability predictions at an early stage in design and would establish a framework for continuous updating of these predictions each time additional information of any of these types is available. An approach based on Bayesian statistical analysis would be uniquely suited to accomplish these goals.

Recommended Approach to Reliability

The following recommendations with regard to reliability analysis result from an analysis of the entire design, development, and testing process. The recommended approach represents an attempt to address the problems just discussed and to provide an approach sufficiently structured so that quantitative measures of reliability based on various types of information can be obtained, and so that these estimates can be updated as additional information becomes available.

The approach, a distillation of various approaches sug-

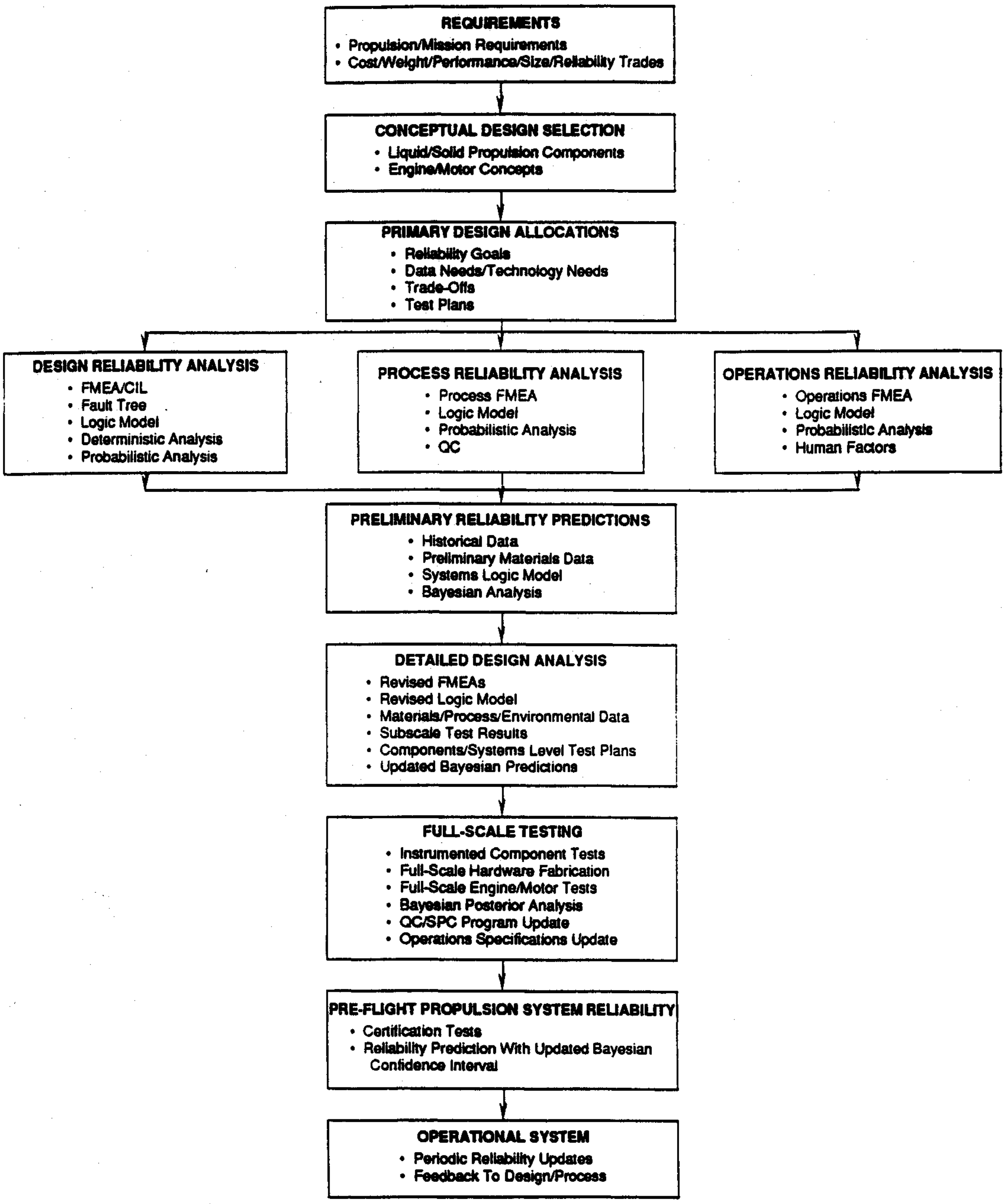

Fig. 1 Reliability approach overview.

gested by the contractors, attempts to combine the strengths of each and to address any perceived remaining weaknesses. It is felt that the overall approach recommended is realistic and doable.

The approach requires the quantification of reliability at the earliest possible stages of design and development. It requires analysis in all phases—design, product development, manufacture, and operations. In fact, it addresses the problem of identifying and solving major reliability issues in design, manufacture, and operations at the initial stages of a project. It emphasizes the probabilistic approach to reliability analysis, identifies analytical methods that are to be employed, identifies data sources for statistical analysis, and recommends statistical methodology. The approach allows for a wide latitude with regard to statistical assumptions, and provides a framework for combining data and other information from many sources to obtain continuous reliability predictions and estimates.

Overview of Recommendations

A summary flowchart of the recommended approach is given in Fig. 1. A detailed flowchart of the entire process, along with a discussion of each item in the chart, have been prepared and are available from the Air Force Astronautics Laboratory. Activities are grouped in the usual DDT&E sequence—conceptual design, preliminary design, and so forth, ending with an operational system. More important, note that design, process, and operations reliability analyses have been listed as separate, though related, functions in the chart. This is to emphasize the importance of analysis in identifying all failure causes.

Beginning with customer-specified design requirements, trade studies are performed to select a basic design concept. In the process, design goals and ground rules regarding cost and reliability are established. Allocations are made at the subsystem, component, and part levels. Relative estimates of system reliability are provided for each candidate design.

After review, a preliminary design is established for the selected propulsion system concept. Reliability goals are reevaluated. Information and technology shortfalls are identified and experiments to address these needs are designed. At this point, the first detailed reliability analysis is undertaken. Design, process, and operations reliability are all included in the analysis. A preliminary system reliability logic model is formulated. FMEA and FTA are used to identify potential failures and their causes. A deterministic analysis is done on all parts, and a preliminary probabilistic analysis is done on all critical parts and/or failure modes. The preliminary probabilistic analysis will provide an initial estimate of reliability based on relatively simple failure models, for example, stress/strength analysis.

In the preliminary probabilistic analysis envisioned, all information (historical data, recent test results, judgment, etc.) is quantified and probability distributions of all relevant variables are specified. These will form the basis of a Bayesian statistical analysis in which a prior distribution (prior to obtaining actual test data), expressing all of this information probabilistically, is formulated. This prior distribution serves as a basis for incorporating (through Bayes' Theorem) additional information as it becomes available from test results at various levels and from other sources. Also, it can be used to obtain a Bayesian confidence interval on part reliability at any stage and, along with the system logic model, forms the basis from which the prior distribution and Bayesian confidence interval on system reliability may be obtained (by computer simulation, if necessary).

In detailed design, the analysis is further refined and updated. Revised FMEAs, FTAs, and logic models are prepared, and newly acquired test data and other information are introduced. At this stage, detailed probabilistic analyses are performed on critical parts/failure modes. This analysis requires the development of realistic failure models that relate failure phenomena to materials properties, geometries, environmental variables, and so forth, and usually requires a modest to substantial amount of computer simulation. Conceptually, all of this information can be used to update the Bayesian prediction model, which then ultimately forms the basis of an updated Bayesian confidence interval for system reliability.

Component and system level test plans are also formulated during the detailed design process. Experimentation is conducted at increasingly higher levels up to full scale testing. In principle, the Bayesian model can be used to determine the number of tests needed to achieve a given reliability objective, as well as allocation to the various levels of testing, through use of the updated prior distributions at any stage. As test data are obtained, they are used to form posterior distributions of part, component, and system reliabilities. Again, these are used as priors for additional test results as well as to obtain Bayesian confidence intervals at each stage. The process can be continued to obtain reliability updates with confidence intervals throughout the operational life of the system.

Additional details on the preparation of FMEAs, CILs, FTAs, and reliability logic models and on the deterministic, preliminary, and detailed probabilistic analyses are given in Refs. 5, 7, and 8. It should be noted that much of what is recommended, at least through preliminary probabilistic analysis, is already being done, at least informally, in the rocket propulsion industry. The suggested approach requires that the analyses be formalized and structured in the context of a Bayesian statistical model.

Probabilistic Analyses

The calculation of system reliability depends on the structure of the system and the interrelationships among its components. This is specified in a reliability logic model. A detailed reliability logic model, in the form of a block diagram, should be prepared for the preliminary design and updated as the design evolves. The model provides a basis for the calculation of system reliability as a function of component and part reliabilities and must therefore adequately model the system in terms of series and parallel connections, levels, and types of redundancies, and so forth. To express system reliability mathematically, it is also necessary to determine interdependencies (coupling) at all levels and, ultimately, to express the exact nature of these interdependencies in probabilistic terms (e.g., in terms of multivariate joint probability distributions, various conditional probabilities, or correlation coefficients). To assure that all failure modes are properly accounted for, fault trees and FMEAs must be consulted in detail in preparing the logic diagram.

The remaining quantities needed for calculating system reliability are the (estimated, predicted, or assumed) reliabilities of each element of the system. At early stages of design and development, many of these reliabilities may be determined, on the basis of overdesign, to be essentially one. Any reliabilities for which this is a doubtful assumption or that are deemed to be critical should eventually be analyzed in considerably more depth. For this purpose, preliminary probabilistic analysis may be used in initial stages of design, and detailed probabilistic analysis may be used as necessary in later stages. In the recommended approach, both must be structured in a Bayesian framework.

Preliminary Probabilistic Analysis

Preliminary probabilistic analysis is defined as any method of analysis leading to a mathematical expression that may be used as a first approximation for reliability. Such an analysis will typically be based on simplified assumptions, dominant failure modes, worst case analyses, tractable probability distributions, data from past programs, and so forth.

We shall consider a preliminary probabilistic analysis based on the traditional stress-strength model. The analysis begins with definitions of stress s and strength S variables. These are ordinarily functions of a number of input variables, whose

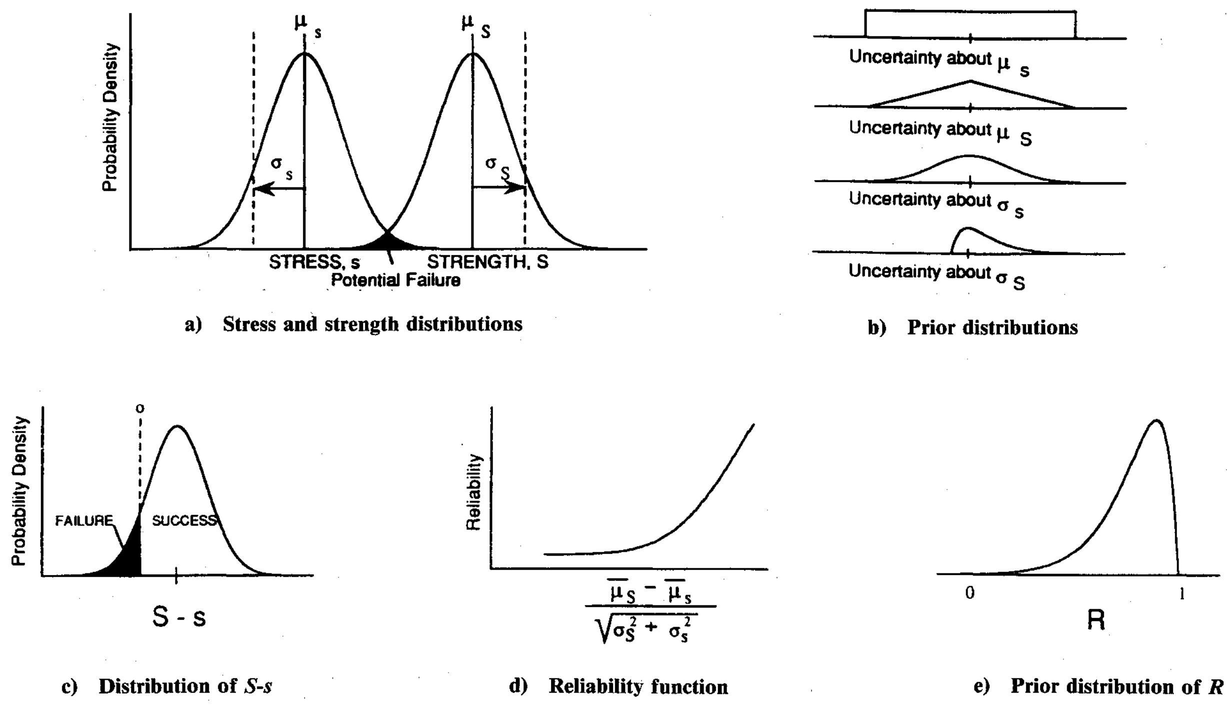

Fig. 2 Stress-strength preliminary probabilistic reliability analysis.

distributions may or may not be known. In any case, it is assumed that the distributions of s and S can be determined or approximated, their parameters specified, and the distribution of S-s derived. The analysis then proceeds in the usual way to the calculation of part reliability.

To implement the Bayesian formulation of the stress-strength analysis, it is necessary to specify, in addition, prior distributions of the parameters of the stress-strength model. This is illustrated in Fig. 2. Here it is assumed that S and s are normally distributed with parameters μ_S, σ_S and μ_s, σ_s, respectively (Fig. 2a). The prior distributions of these parameters are determined from past experience, judgment, and so forth. Representative priors are shown in Fig. 2b.

Given values of the parameters, the distribution of S-s is normal with mean $\mu = \mu_S - \mu_s$ and standard deviation $\sigma = [\sigma_S^2 + \sigma_s^2]^{1/2}$, as shown in Fig. 2c. Part reliability R, calculated as $R = P(S\text{-}s > 0)$, is shown as a function of μ and σ in Fig. 2d. Finally, the priors on the μ and σ impose a probabilistic structure on R, leading to the prior distribution on part reliability in Fig. 2e. How these are used in a Bayesian analysis will be discussed later.

Detailed Probabilistic Analysis

Detailed probabilistic analysis must be done for all critical failure modes and others that cannot be adequately modeled by the stress-strength approach or other simple failure models. Typical failure modes of this type are high-cycle fatigue, low-cycle fatigue, debonding, flaw propagation, bearing wear, corrosion, and seal leakage. The analysis requires the development of variable transformations, failure models, and other complex engineering models. They usually involve computer simulations employing finite elements routines, Monte Carlo techniques, and careful statistical analyses of the results. Important elements of this approach are given in Fig. 3. A detailed description of the method is given in Ref. 7 and in the articles cited therein.

Detailed probabilistic analysis requires that probability distributions be specified at the input-variable level. Historical data, specific laboratory and test data, and engineering analysis may be used in conjunction with engineering judgment to assist in determining the appropriate distributions. Note that the available information is quantified at the most elemental level, in terms of the life drivers, as opposed to the more complex levels (e.g., stress). Thus, judgment and experience are employed at the level at which they are most likely to be valid, relevant, and reliable.

To simulate the random failure process, it is necessary to relate the process to the life drivers using the life driver transformation. The form of the transformation depends on the specific application, i.e., on the failure mode being analyzed. A statistical synthesis of the engineering analysis is required to extract the life drivers. In simulating the failure mechanism, one must account for possible model inaccuracies. Thus,

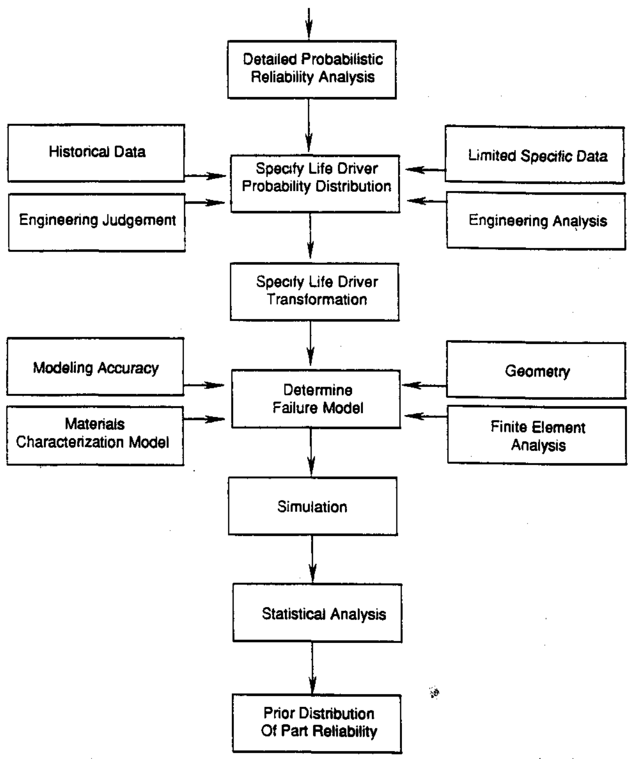

Fig. 3 Detailed probabilistic reliability analysis flow chart.

model misspecifications, uncertainties in model parameters, and so forth, must also be included in the analysis.

Material properties may be characterized in several dimensions and as functions of various input variables. Models expressing these relationships may be important elements of the failure model. Uncertainties regarding these relationships are also included in the analysis. Geometrical configurations, points of maximum stress, dimensions, etc., are included as appropriate to the specific application.

A finite elements analysis may be used in computer simulations to model the interrelationships between various stresses and other characteristics of complex geometrical configurations that cannot be investigated analytically. The failure model is estimated by means of Monte Carlo simulation through as many cycles as necessary to characterize the process adequately and statistically relate the failure distribution to the many input variables and their distributions, the uncertainties in models, parameters, judgment, and any other potentially important factors that may be identified. The process will ordinarily involve a number of steps, e.g., selection of input distributions, random selection of parameters of each distribution, random selection of values for each variable, calculation of values of transformed variables, and so forth.

Analysis of data plays an important role in many areas of the reliability assessment process. Here, statistical analysis of the simulation results is required to structure the simulated failure data and investigate the relationships of the results to the various input variables, assumptions made, and other factors. The output will be an empirical distribution that will form the basis of the Bayesian prior on part reliablity for these parts.

Bayesian Reliability Analysis

The statistical foundation of the analytical approach suggested is Bayesian statistical analysis. This requires the specification of prior distributions for basic inputs to the analysis and thereby provides a mechanism for quantification of judgmental information. Once the priors are specified, the Bayesian analysis provides a mechanism for combining this information with test results to obtain a posterior distribution of reliability. The methodology for Bayesian analysis is provided for many basic reliability problems by Martz and Waller.[9]

The proposed approach will require Bayesian updating at many stages of the analysis. The end result at each stage is a Bayesian confidence interval that incorporates all available information and provides a measure of belief with regard to the level of reliability attained at that stage.

Prior information may be specified with regard to basic materials, structures, and environments, at the part, component, and system level. A methodology for combining the information from these various and diverse sources is given by Martz et al.[10,11] The method as presented is for series systems only of binomial components in Ref. 10; it is extended to parallel and combinations of series and parallel connections of binomial components in Ref. 11. In analyzing more complex systems, further extensions of the theory will be necessary. It is anticipated, in any case, that simulations (perhaps extensive) will be required in most realistic applications.

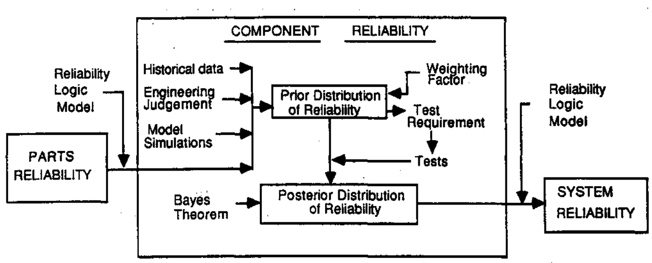

Fig. 4 Bayesian reliability analysis.

A schematic representation of this process is presented in Fig. 4 for calculation of component reliability. It is assumed that prior distributions on part reliability have been obtained by methods such as those described in the preceding sections. These are combined by means of the reliability logic model to form an induced prior distribution on component reliability. In addition, other information may be available at this level as well. Weights are assigned to these two types of information to reflect their relative importance and a composite prior is formed. (Martz et al.[10,11] approximate the derived distributions at several stages of the analysis by equating moments to those of mathematically tractable forms.)

In principle, the results may be used at a certain point to determine sample size requirements for ensuing test programs. Upon completion of the tests, Bayes' Theorem is used to obtain the posterior distribution of component reliability. The whole process is now repeated at the system level, again combining component reliabilities through the system logic model to form an induced prior for system reliability. Weights are again specified, a composite prior is formed, test data are obtained, the posterior distribution of system reliability is determined, and a Bayesian confidence interval on system reliability is calculated.

Sample Size Determination

It is well known that, with ordinary binomial sampling, 230 tests without failure are necessary to demonstrate an item reliability of 0.99 with 90% confidence. This is equivalent, in the Bayesian context, to beginning with a uniform prior distribution, i.e., assigning uniform probability density to the entire interval (0,1). Using a uniform prior says that one knows nothing about the true reliability R and feels that it is equally likely to be any of its possible values.

Of interest is the question of how large n, the number of tests required, would be with other choices of a prior distribution, for example, one that specified that R was much more likely to be near 1 than near 0. This question may be approached as follows. In the case of the binomial model (success/fail data), the natural prior to use is the beta distribution. This, in fact, forms the basis of the Martz et al.[10,11] approach. Once the beta prior is determined, one can determine the required sample size by assuming a fixed number of failures and looking at the posterior distribution (also a beta distribution[9]) as a function of n. The required n is found by trial and error, increasing the value of n in the computation of confidence until the reliability goal (e.g., 0.99 at 90% confidence) is achieved. (The computations can be done using most standard statistical packages; Minitab was used in the calculations that follow.)

The beta distribution is determined by two parameters, a and b, with mean M and variance V given as functions of these parameters by the relationships $M = a/(a + b)$ and $V = ab/[(a + b)^2(a + b + 1)]$. In specifying priors, one may specify values for M and V and then solve these relationships for a and b. Alternatively, a and b may be determined from previous outcomes or other information.

As an example, consider the liquids data reported earlier with $n = 1380$, $X = 1363$, and $R = 0.9877$. The corresponding parameters for the beta distribution are $a = 1364$ and $b = 18$. Even with this large a prior sample, with so many failures, many additional tests are required to achieve an increase in reliability with a high degree of confidence. In Table 3, the confidence that the reliability is at least 0.99 is given as a function of the number of additional tests n required, with no additional failures. From this, we see that 90% confidence in 0.99 reliability is obtained only after 980 additional tests with no failures. If all of the liquids data are used (i.e., $n = 1522$, $X = 1500$, and $R = 0.9855$), the result is even worse; over 1400 tests with no failures would be required.

Another way of representing outcome data is to argue that the problems leading to failure in earlier cases have all been fixed and that, for example, 0.9855 is equivalent to what

Table 3 Confidence that reliability is at least 0.99 vs sample size n, beta prior with $a = 1364$, $b = 18$

n	Confidence
50	0.19
100	0.23
150	0.28
200	0.32
250	0.37
300	0.42
350	0.47
400	0.52
450	0.57
500	0.61
550	0.66
600	0.70
650	0.73
700	0.77
750	0.80
800	0.83
850	0.85
900	0.87
950	0.89
1000	0.90

Table 4 Confidence that reliability is at least 0.99 vs sample size n, beta prior with $a = 68$, $b = 1$, and number of failures $f = 0, 1, 2$, and 3

n	$f = 0$	$f = 1$	$f = 2$	$f = 3$
50	0.67	0.34	0.12	0.04
100	0.82	0.51	0.25	0.10
150	0.89	0.65	0.37	0.19
200	0.94	0.75	0.50	0.29
250		0.83	0.62	0.40
300		0.89	0.71	0.51
350		0.92	0.79	0.61
400			0.85	0.70
450			0.89	0.77
500			0.92	0.82
550				0.87
600				0.90

would have resulted from N tests with no failures. Since the mean of the beta distribution is $a/(a + b)$ and zero failures is equivalent in Bayesian analysis to $b = 1$, this suggests that prior information be represented, in this example, by a beta distribution with $a = 68$ and $b = 1$. The confidence in 0.99 reliability in that case is tabulated as a function of n in Table 4. In this table, confidence as a function of n is given assuming 0, 1, 2, and 3 failures in the additional tests. In these cases, the number of additional tests required for 90% confidence in 0.99 reliability are 161, 319, 461, and 596, respectively.

This illustrates a general class of difficulties encountered in the Bayesian approach, namely, selection of appropriate prior distributions to represent whatever information is available at a given time. It is generally desirable to have dispersed priors (i.e., priors with relatively large standard deviations) to allow for shifts in the posterior when the data suggest this. Two cautions must be observed in applications in propulsion system reliability.

1) It is difficult to introduce much dispersion and still have a sensible prior if the prior R is a very high number (e.g., 0.999, 0.9999, etc.). This therefore becomes a particularly important problem to be dealt with below the system level.

2) Significant shifts in the posterior may indicate that the modeling efforts in formulating the priors were inadequate. The models should be looked at again very carefully. Note, too, that sensitivity studies are essential in these applications.

Implementation

A number of revisions and extensions of existing Bayesian methodology may be necessary to apply Bayesian reliability analysis to rocket propulsion systems. With this in mind, the following steps are recommended for implementing the approach described in this paper.

1) Formulate system reliability model.
2) Analyze relevant historical data at the part, component, and system levels.
3) Perform preliminary reliability allocation.
4) Formulate prior distributions at each level based on available data, simulation results, and judgment.
5) Calculate reliability predictions and compare with allocations.
6) Iterate design as necessary.
7) Determine sample size requirements to achieve allocated reliability with specified confidence.
8) Perform sample size tradeoffs between levels of testing.
9) Determine optimal test allocation plan.
10) Investigate sensitivity of solution to prior distributions, models, allocation, and other factors.
11) Determine weighting functions.
12) Determine posterior distributions.
13) Complete Bayesian analysis.

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